Name: ____________________________ Group ______

1. A spinner has the left side (numbers 1, 2, 3, 4, and 5) colored red and the right side colored white (numbers 6, 7, 8, and 9), with all numbers equally likely.

a) What is the probability the spinner lands on an odd number?

b) Given that the spinner landed on an odd number, what is the chance the spinner landed on a white number?

c) Given that the spinner landed on a white number, what is the chance it landed on an odd number?

d) Is the spinner landing on a white number independent of the spinner landing on an odd number?

3. Jack and Jill are independently struggling to pass their last (one) class required for graduation. Jack needs to pass Calculus III, but he only has a probability of 0.30 of passing. Jill needs to pass Advanced Pharmaceuticals, but she only has a probability of 0.46 of passing.

a) What is the probability that both get their diploma?

b) What is the probability that neither get their diploma?

c) what is the probability that at least one gets a diploma?

2. Data on the marital status of U.S. adults can be found in *Current Population Reports*, a publication of the U.S. Bureau of the Census. The following table provides a joint probability distribution for the marital status of U.S. adults by sex. We have used "Single" as an abbreviation for "never married." If a U.S. adult is selected at random, determine the probability that

	Single (M_1)	Married (M_2)	Widowed (M_3)	Divorced (M_4)	Total
Male (S_1)	0.129	0.298	0.013	0.040	0.480
Female (S_2)	0.104	0.305	0.057	0.054	0.520
Total	0.233	0.603	0.070	0.095	1.000

a) the adult selected is divorced, given that the adult selected is a male.

b) the adult selected is a male, given that that adult selected is divorced.

c) Why is the answer to b) greater than the answer to a)?

4. A class of 40 students has 17 males and 23 females. Two students are selected at random. Find the probability that

a) the first student is a male.

b) the first student is a female and the second student is a male.

c) the first student is a male and the second student is a male.

d) The second student is a male.

e) What is the relationship between parts a) and d)? Is this a coincidence? Explain your answer.

The following are some proofs. I will give the question on the front of the page and the worked out answer on the back.

5. Prove Theorem 2.19: $P(A^c) = 1 – P(A)$. Each step must have an explanation of either one of Axioms or a definition. No theorems are allowed.

6. Prove Theorem 2.20: If $A \subset B$ then $P(A) \leq P(B)$. Each step must have an explanation of either one of Axioms or a definition. No theorems are allowed.

7. Prove Part of Theorem 3.20: If A and B are independent, then A^C and B are independent. Each step must have an explanation of either one of Axioms or a definition. No theorems are allowed.

8. Prove Theorem 2.33: $P(A \cup B \cup C) = P(A) + P(B) + P(C) – P(A \cap B) – P(B \cap C) – P(A \cap C) + P(A \cap B \cap C)$

5. Prove Theorem 2.19: $P(A^c) = 1 - P(A)$. Each step must have an explanation of either one of Axioms or a definition. No theorems are allowed.

Step	Reason
$1 = P(S)$	completeness axiom
$= P(A \cup A^C)$	definition of complement
$= P(A) + P(A^C)$	definition of complement: A and A^C are disjoint
	additivity axiom
$P(A^C) = 1 - P(A)$	algebraic rearrangement

6. Prove Theorem 2.20: If $A \subset B$ then $P(A) \leq P(B)$. Each step must have an explanation of either one of Axioms or a definition. No theorems are allowed.

Step	Reason
$P(B) = P(A \cup B \backslash A)$	definition of $A \subset B$
$= P(A) + P(B \backslash A)$	definition of setminus: A and $B \backslash A$ are disjoint
	additivity axiom
$P(B) \geq P(A)$	nonnegativity axiom

7. Prove Part of Theorem 3.20: If A and B are independent, then A^C and B are independent. Each step must have an explanation of either one of Axioms, definition or a theorem.

Need to show that $P(A^C \cap B) = P(A^C)\, P(B)$.

Step	Reason
$P(A^C)\, P(B) = (1 - P(A))P(B)$	complement rule
$= P(B) - P(A)P(B)$	algebra
$= P(B) - P(A \cap B)$	independence of A and B
$= P(B \backslash A)$	definition of setminus
$= P(B \cap A^C)$	definition of setminus (aditivity)

8. Prove Theorem 2.33: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Step	Reason
$P(A \cup B \cup C) = P(A \cup (B \cup C))$	associative rule
$= P(A) + P(B \cup C) - P(A \cap (B \cup C))$	inclusion – exclusion for 2 events
$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap (B \cup C))$	inclusion – exclusion for 2 events
$= P(A) + P(B) + P(C) - P(B \cap C) - P((A \cap B) \cup (A \cap C))$	distributive rule
$= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap A \cap C)]$	inclusion – exclusion for 2 events
$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap A \cap C)$	algebra
$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$	def of intersection ($A \cap A = A$)